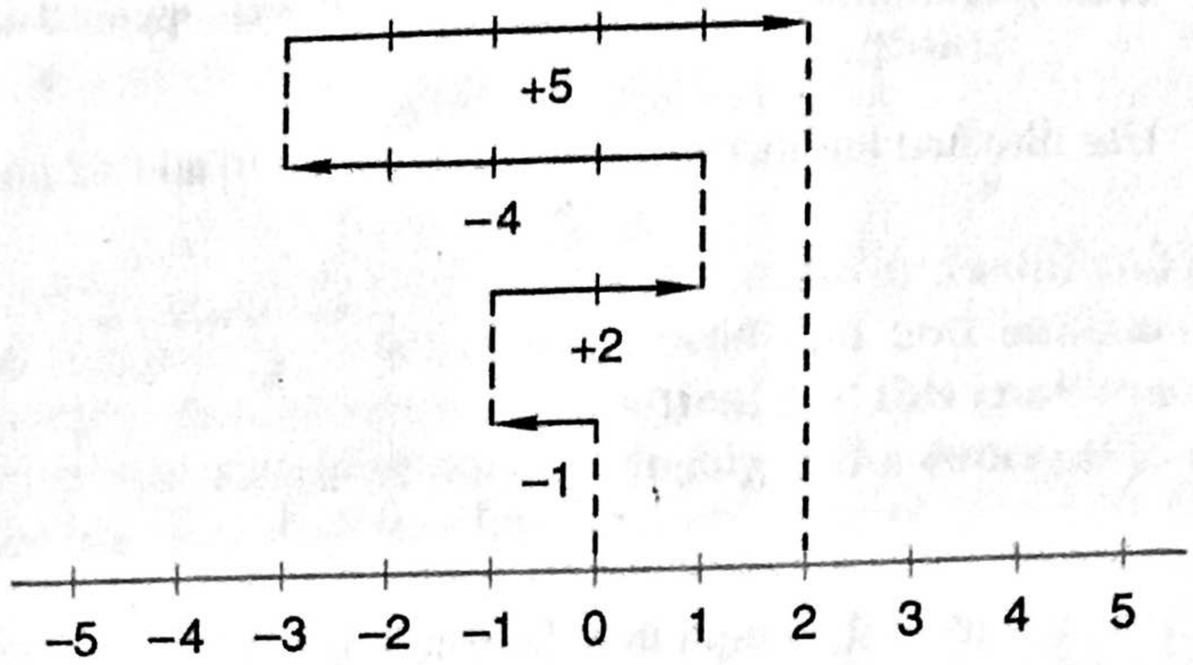
The answer will be the same regardless of the order in which we draw the arrows. To show this, we will work the problem again with the order of the numbers changed.

$$(-1) + (+2) + (-4) + (+5)$$



Again we find that the sum of the numbers is +2.

practice Simplify:

a. $|-4|$ **b.** $|4.2|$ **c.** $-|10 - 6|$ **d.** $-|-14 + 6|$

Draw a number line for each of the following problems and use directed numbers (arrows) to add the signed numbers.

e. $(+3) + (+2)$

f. $(-3) + (+2)$

g. $(-5) + (+2) + (-3) + (+3)$

problem set 5

1. (5) (a) Use braces and digits to designate the set of natural numbers.

(b) Use braces and digits to designate the set of whole numbers.

(c) Use braces and digits to designate the set of integers.

2. (4) What do we call the point on the number line with which we associate the number zero?

3. (4) (a) What is the graph of a number?

(b) What is the coordinate of a point on the number line?

(c) How can we tell if one number is greater than another number?

Simplify:

4. (5) $|-8|$ **5.** (5) $|+8|$ **6.** (5) $|-12|$

7. (5) $-|15 - 5|$ **8.** (5) $-|-15 + 5|$ **9.** (5) $|12 - 30|$

Draw a number line for each of the following problems and use directed numbers (arrows) to add the signed numbers.

10. (5) $(+3) + (-8)$ **11.** (5) $(-1) + (+2)$

12. (5) $(+4) + (+3)$ **13.** (5) $(-4) + (+2) + (-4) + (+8)$

14. (4) Use one unit multiplier to convert 28 centimeters to inches (2.54 cm = 1 in.).

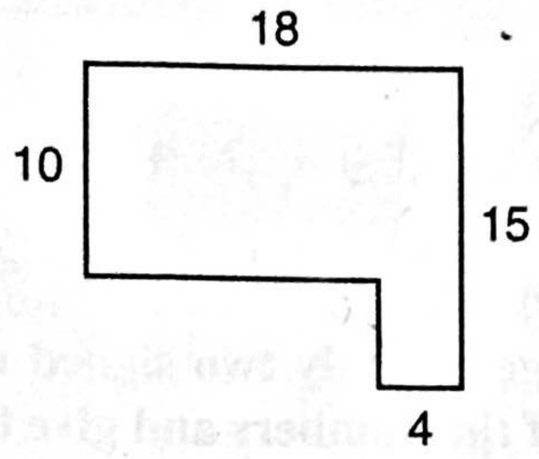
15. (4) Use two unit multipliers to convert 42 centimeters to feet. (Go from centimeters to inches to feet.)

16. (3) The length of a rectangle is 22 inches. The width of the rectangle is 13 inches. Find the perimeter of the rectangle.

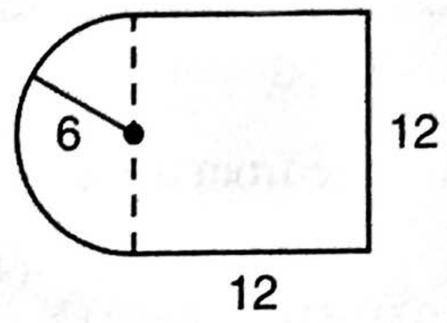
17. (3) The radius of a circle is 10 feet. Find the circumference of the circle.

Find the perimeter of each figure. Corners that look square are square. Dimensions are in yards.

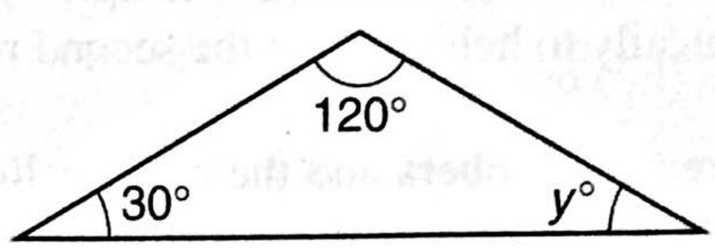
18. (3)



19. (3)



20. (2) Find y.



Add, subtract, multiply, or divide as indicated:

21. (1) $6\frac{2}{3} + 7\frac{4}{9}$

22. (1) $95\frac{1}{8} - 4\frac{13}{16}$

23. (4) $4\frac{1}{2} \times 2\frac{2}{3}$

24. (4) $4\frac{1}{2} \div 7\frac{3}{8}$

25. (4) $\dfrac{7\frac{1}{8}}{4\frac{2}{5}}$

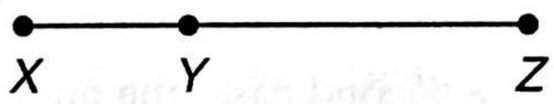
26. (4) $23.0106 + 0.1094$

27. (4) $48.2 - 13.34$

28. (4) 8.08×0.120

29. (4) $8.48636 \div 2.12$

30. (1) XZ is $18\frac{2}{5}$ miles. XY is $6\frac{1}{15}$ miles. Find YZ.



LESSON 6 Rules for Addition • Adding More Than Two Numbers • Inserting Parentheses Mentally • Definition of Subtraction

6.A rules for addition

In the preceding lesson we learned to add signed numbers by using a number line and arrows to represent the numbers. This procedure allows us to have a graphical picture of what we are doing. Unfortunately, this method is slow and time-consuming. We do not have time to go through the entire algebra course drawing number lines and arrows, so we must develop rules that will allow us to do algebraic addition quickly. We need two rules—one to use when the numbers to be added have the same signs and one to use when the numbers have different signs. In the following example we will draw two diagrams that will help us state the first rule.

example 6.1 Use directed numbers and the number line to add +1 and +3 algebraically, and use directed numbers and the number line to add –1 and –3 algebraically.

example 6.6 Simplify: $-2 + 11 - 4 + 3 - 2$

solution We mentally enclose the numbers in parentheses and add algebraically to get a sum of +6.

$$(-2) + (+11) + (-4) + (+3) + (-2) = \mathbf{+6}$$

example 6.7 Simplify: $(-4) + |-2| + 3 - 7 - 2$

solution We mentally insert parentheses so that the problem reads as follows:

$$(-4) + (|-2|) + (+3) + (-7) + (-2)$$

Now we simplify and get an answer of –8.

$$(-4) + (+2) + (+3) + (-7) + (-2) = \mathbf{-8}$$

6.D definition of subtraction

As we have seen, if we use algebraic addition, we can handle minus signs without using the word *subtraction*. We let the signs tell whether the numbers are positive or negative, and we mentally insert parentheses and extra plus signs as necessary. Thus the subtraction problem on the left

$$7 - 4 = 3 \qquad 7 + (-4) = 3$$

can be turned into the algebraic addition problem on the right. A definition of algebraic subtraction does exist, however, and some people prefer to use it rather than using mental parentheses. The result is exactly the same, but the definition uses the word *subtraction*. To subtract algebraically, we change the sign of the subtrahend and add.

$$7 - 4 = 3 \qquad 7 + (-4) = 3$$

The formal definition of the operation of algebraic subtraction is as follows.

ALGEBRAIC SUBTRACTION

If a and b are real numbers, then

$$a - b = a + (-b)$$

where $-b$ is the opposite of b.

Thus there are two thought processes that may be used to simplify expressions that contain minus signs such as

$$7 - 4$$

Since we prefer to consider that the minus sign designates a negative number, we will emphasize algebraic addition in this book and will avoid the use of the word *subtraction*.

practice Use parentheses to enclose each number or expression and its sign. Then insert plus signs between the parentheses. Then add to get a sum.

a. $-5 - 2 + 7 - 6$

b. $-4 - |-2| - 6 + (-5)$

c. $-|-8| - 3 + 5 - 11$

d. $-8 + |-6| - |5| - 7$

problem set 6

1. (6) State the rule for adding two numbers whose signs are alike.

2. (6) State the rule for adding two numbers whose signs are different.

3. (5) What property of addition states that the order in which two real numbers are added does not affect the sum?

4. (4) (a) What do we call the answer to an addition problem?

(b) What do we call the answer to a subtraction problem?

(c) What do we call the answer to a multiplication problem?

(d) What do we call the answer to a division problem?

Simplify:

5. (5) $|-5|$

6. (5) $-|10 - 7|$

7. (5) $|3 - 6|$

Draw a number line for each of the following problems and use directed numbers (arrows) to add the signed numbers.

8. (5) $(+4) + (-5)$

9. (5) $(+1) + (-3) + (+4) + (-2)$

10. (4) Use one unit multiplier to convert 34 meters to centimeters (100 cm = 1 m).

11. (4) Use two unit multipliers to convert 6 miles to inches. (Go from miles to feet to inches.)

12. (3) The perimeter of a square is 36 centimeters. What is the length of one side of the square?

13. (3) The diameter of a circle is 14 meters. Find the circumference of the circle.

Simplify:

14. (6) $(+3) + (-14)$

15. (6) $(-3) + (-14)$

16. (6) $(-5) + (4) + (-3) + (+8)$

17. (6) $(-3) + (+2) + (-2) + |-2|$

Use parentheses to enclose each number and its sign. Then insert plus signs between the parentheses. Then add to get a sum.

18. (6) $-2 + 11 - 4 + 3 - 8$

19. (6) $-5 - 11 + 20 - 14 + 5$

20. (6) $-4 - 3 + 2 - 4 - 3 - 8$

21. (6) $7 - 3 + 2 - 11 + 4 - 5 + 3$

Use parentheses to enclose each number or expression and its sign. Then insert plus signs between the parentheses. Then add to get a sum.

22. (6) $-7 + (-8) + 3$

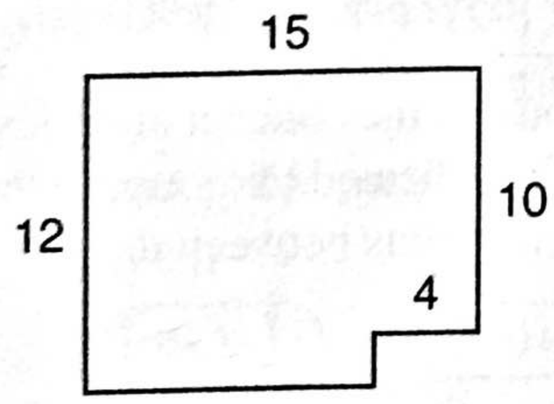
23. (6) $-7 + (-3) + 4 - 3 + (-2)$

24. (6) $-4 - 2 + (+8) + |-5|$

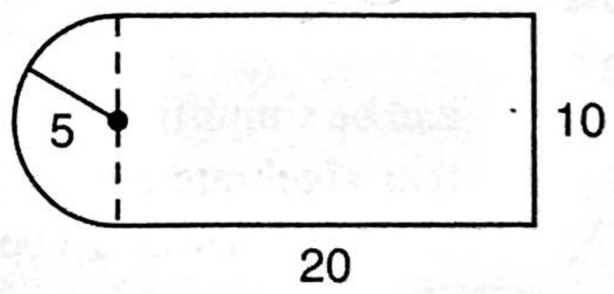
25. (6) $+|-2 - 3| - 4 + (-8)$

Find the perimeter of each figure. Corners that look square are square. Dimensions are in kilometers.

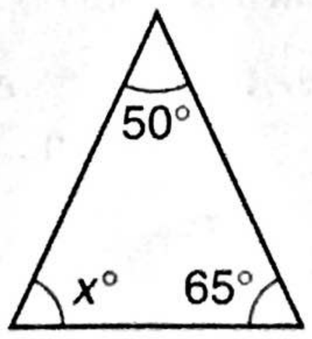
26. (3)



27. (3)

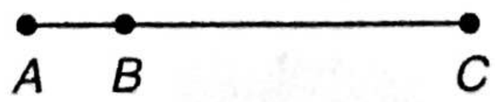


28. (2) Find x.



29. (4) Simplify: $\frac{21}{5} \times \frac{15}{7} \times \frac{4}{9}$

30. (1) AC is $25\frac{3}{4}$ inches. BC is $20\frac{1}{20}$ inches. Find AB.



solution We see three numbers are to be added. We begin by enclosing each number and inserting the necessary plus signs.

$$\text{(}-(-3)\text{)} + \text{(}-[-(-2)]\text{)} + \text{(}+[-(-3)]\text{)}$$

Now we simplify within each enclosure and then add.

$$(+3) + (-2) + (+3) = \mathbf{4}$$

example 7.3 Simplify: $-(-4) + (-2) - [-(-6)]$

solution This time we will picture the enclosures mentally but we will not write them down. If we do this, we can simplify the given expression as

$$(+4) + (-2) + (-6) = \mathbf{-4}$$

example 7.4 Simplify: $-(+4) - (-5) + 5 - (-3) + (-6)$

solution This time we will not even use parentheses but will write the simplification directly as

$$-4 + 5 + 5 + 3 - 6 = \mathbf{3}$$

It might take a lot of practice to become adept in doing simplifications such as this one. Do not get discouraged if you find these problems troublesome.

practice Use the concept of opposites and algebraic addition to simplify the following. Use additional plus signs and brackets as required.

a. $-(-3) - (-4)$ **b.** $+(-5) + [-(-6)]$

c. $-(+6) - (-8) + 7 - (-3) + (-5)$ **d.** $-(-3) - [-(-4)] + [-(-6)]$

problem set 7

1. (6,7) (a) What is the opposite of 2?

(b) What is the opposite of –2?

(c) What is the sum of a real number and its opposite?

2. (6) What is another name for the opposite of a number?

3. (5) (a) Designate the set of natural numbers.

(b) Designate the set of whole numbers.

(c) Designate the set of integers.

Use the concept of opposites to simplify:

4. (7) $-(+4)$

5. (7) $-(-4)$

6. (7) $-[-(-4)]$

7. (7) $-\{-[-(-4)]\}$

8. (4) Use one unit multiplier to convert 2200 centimeters to meters (100 cm = 1 m).

9. (4) Use two unit multipliers to convert 3000 inches to miles. (Go from inches to feet to miles.)

10. (3) The length of a rectangle is 32 inches. The width of the rectangle is 16 inches. Find the perimeter of the rectangle.

11. (3) The radius of a circle is 12 feet. Find the circumference of the circle.

Use the concept of opposites and algebraic addition to simplify the following. Use additional plus signs and brackets as required.

12. (7) $+7 - (-3) + (-2)$

13. (7) $-3 + (-2) - (-3)$

14. (7) $-(-3) - [-(-4)] - 2 + 7$

15. (7) $-2 - (-3) - \{-[-(-4)]\}$

16. (7) $-(-2) - |-2|$

17. (7) $-|-10| - (-10)$

18. (7) $-3 - (-3) + |-3|$

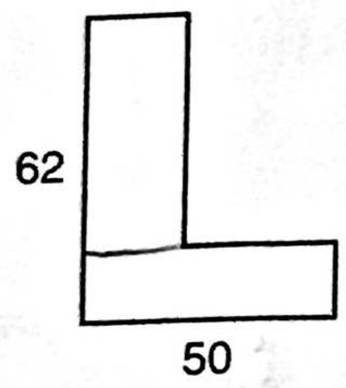
19. (7) $-2 - [-(-6)] + |-5|$

20. (7) $-|-3 - 2| - (-3) - 2 - 5$

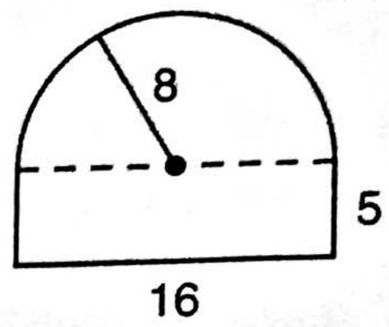
21. (7) $|-2 - 5 - 7| - (-4)$

Find the perimeter of each figure. Corners that look square are square. Dimensions are in yards.

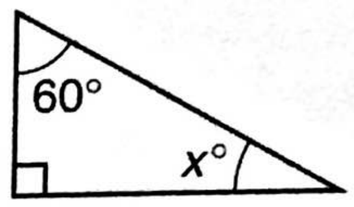
22. (3)



23. (3)



24. (2) Find x.



Add, subtract, multiply, or divide as indicated:

25. (1) $5\frac{1}{2} + 7\frac{3}{8} - 1\frac{1}{4}$

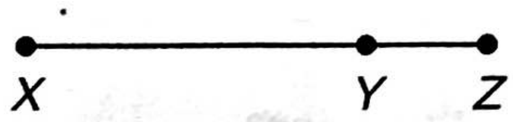
26. (4) $1\frac{3}{5} \times 12\frac{1}{2}$

27. (4) $4\frac{1}{4} \div 3\frac{2}{5}$

28. (4) $0.00143 + 0.012 + 443.6 + 0.0007$

29. (4) 3.628×0.0404

30. (1) The length of $\overline{XY}$ is $16\frac{2}{3}$ centimeters. The length of $\overline{YZ}$ is $5\frac{5}{6}$ centimeters. Find XZ.

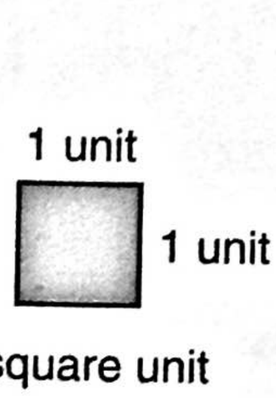


LESSON 8 Area

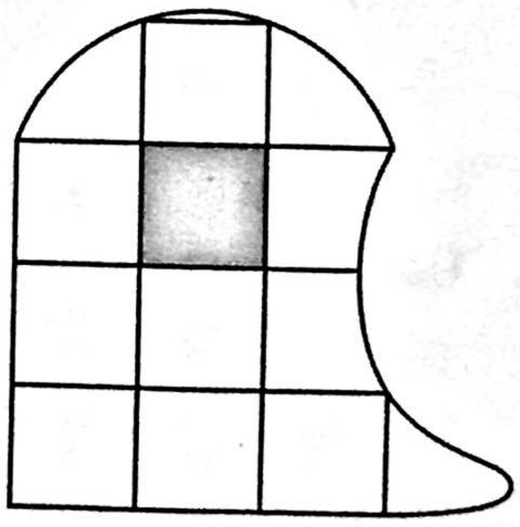
8.A

the concept of area

The **area** of a closed figure is the number of square units contained in the figure. A **square unit** is a square having sides that measure one unit in length. Any shape that fills a closed figure without overlapping can be used to measure area, but the square is used because of its simplicity.



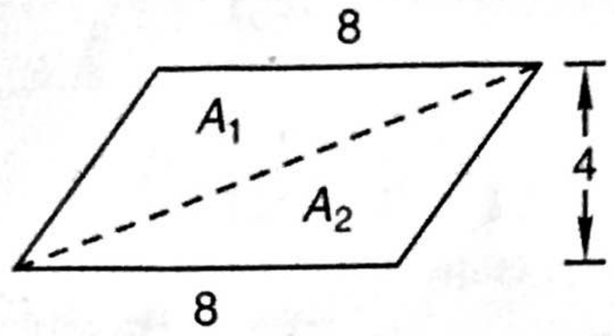
1 square unit



Closed figure with irregular boundary

For many closed figures with irregular boundaries, we must "break up" some of the square units to fill the figure. Therefore, it is not uncommon to have closed figures whose areas contain fractional square units. We can see that counting square units is not the easiest or the best way to find the area of a closed figure. Therefore, in the following sections we will develop formulas for computing the areas of some common geometrical figures.

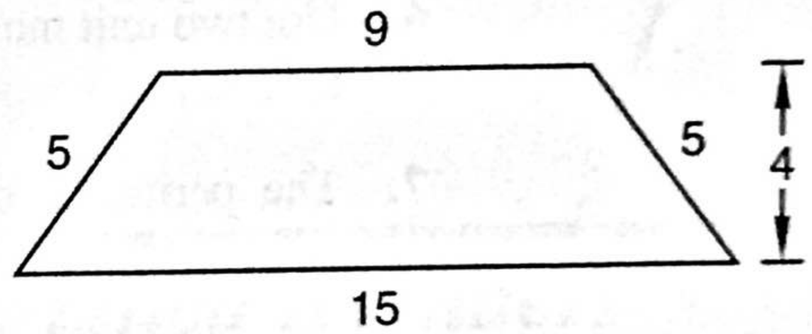
solution A diagonal of a parallelogram divides the figure into two triangles whose areas are equal. The base of each triangle is 8 cm and the height of each triangle is 4 cm.



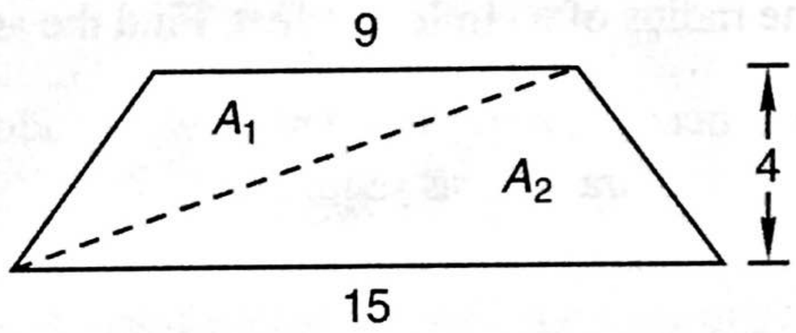
The area of the parallelogram equals the sum of the areas of the two triangles.

$$\text{Area} = A_1 + A_2$$
$$= \frac{1}{2}(8\text{ cm})(4\text{ cm}) + \frac{1}{2}(8\text{ cm})(4\text{ cm})$$
$$= 16\text{ cm}^2 + 16\text{ cm}^2$$
$$= \mathbf{32\text{ cm}^2}$$

example 8.10 Find the area of this trapezoid. Dimensions are in meters.



solution A diagonal of a trapezoid divides the figure into two triangles. The base of one triangle is 9 m and the base of the other triangle is 15 m. The height of each triangle is 4 m.

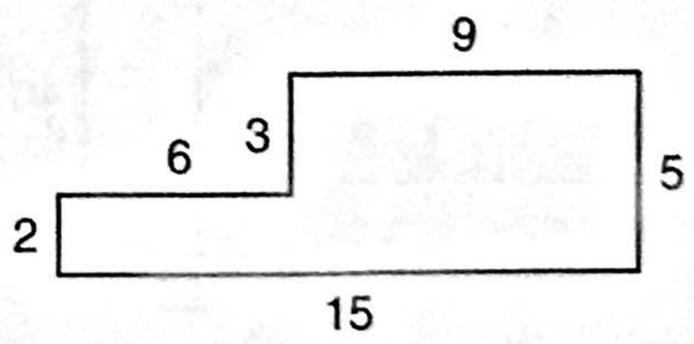


The area of the trapezoid equals the sum of the areas of the two triangles.

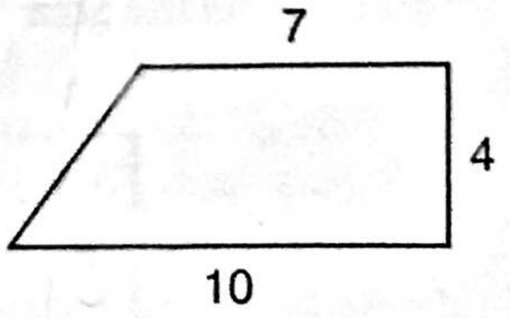
$$\text{Area} = A_1 + A_2$$
$$= \frac{1}{2}(9\text{ m})(4\text{ m}) + \frac{1}{2}(15\text{ m})(4\text{ m})$$
$$= 18\text{ m}^2 + 30\text{ m}^2$$
$$= \mathbf{48\text{ m}^2}$$

practice

a. Find the area of this figure. All angles are right angles. Dimensions are in inches.

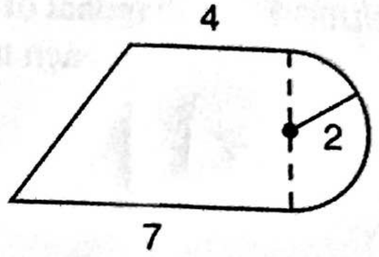


b. Find the area of this figure. Corners that look square are square. Dimensions are in feet.



c. The radius of a circle is 5 centimeters. Find the area of the circle.

d. Find the area of this figure. Corners that look square are square. Dimensions are in meters.



problem set

1. (a) What is the opposite of $\frac{1}{2}$?